Math Propulsion: Engaging Math Learners Through Embodied Performance & Visualization

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ABSTRACT

This paper describes a series of interaction design sketches we created to supplement mathematics curricula. These sketches were deployed in a variety of secondary school math classrooms in the San Francisco Bay Area. The activities purposefully use visualization and embodiment to engage students with the math concepts of geometric transformations and symmetrical patterning. These experiments exemplify how applying embodiment and visualization to traditionally impersonal and abstract subjects like math can make the learning experience more fun and active for students and offer new pedagogical strategies for teachers.

Author Keywords

Embodied, cognition, visualization, mathematics, education

ACM Classification Keywords

H5.m. Information interfaces and presentation (e.g., HCI): Miscellaneous.

General Terms

Design, Experimentation

INTRODUCTION

Mathematics is commonly vilified as the most difficult subject in primary and secondary school curriculum; the educational research literature documents the prevalence of "math anxiety" [1, 11] and "math avoidance" [26] behaviors, with anxiety levels peaking in grades 9 and 10 when students are studying geometry and algebra [16]. This aversion to mathematics is deeply troubling, not only because mathematics capability is the cornerstone to careers in science and technology, but also because we believe, as mathematician and teacher Paul Lockhart states so well, "that there is nothing as dreamy and poetic, nothing as radical, subversive, and psychedelic, as mathematics." [15]

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Figure 1. Superimposing video frames enables students to explore geometric rotations - by making "math angels"

MATH PROPULSION

In this paper, we describe a series of experiments we conducted employing embodied performance and visualization that emphasizes conceptual exploration, collaboration, and creativity over "getting the right answer." These formative experiments share the theme of "Math Propulsion" because they engage students with math using bodily action. This strategy is grounded in the thesis that embodied interactions [6, 8, 14]—those that are tangible, social and also corporal—would increase students' engagement with and understanding of the subject of math.

We expect the use of embodied performance and visualization to contribute to mathematics education in several ways:

1. Physical engagement helps to entail mental engagement as students reason about how their actions are influencing the visual feedback they are receiving.
2. Social engagement and negotiation of joint actions enables students to apply mathematical vocabulary and concepts to an immediate situation, and to engage in criticism and appraisal of mathematical issues.
3. Direct physical control over mathematical variables, coupled with real-time feedback, gives students a chance to discover for themselves how different factors relate to one another; for instance, they can see how

movement of vertices on a polygon affects the area and symmetry of the shape.

4. Mathematics can be seen in a personal and situated context, where the numbers, shapes or equations are part of the immediate world rather than something from a hypothetical and isolated one.
5. Mathematics can be presented as a subject for open-ended inquiry and invention rather than a series of prescribed exercises.

BACKGROUND

This work is motivated by substantial bodies of prior work in education pedagogy, digital learning tools, embodied cognition, and the visual and performing arts. This narrative overview highlights some of our key inspirations and influences.

The Need for New Tools in Mathematics Education

The National Council of Teachers of Mathematics (NCTM) has long promoted the following steps to promote wider interest and acceptance of math [16]: Accommodating for different learning styles; emphasizing the importance of original, quality thinking rather than rote manipulation of formulas; letting students have some input into their own evaluations; designing positive experiences in math classes; and allowing for different social approaches to learning mathematics

These recommendations are based on years of research showing that traditional mathematics education poorly adapts to different learning styles and also fails to address students' spatial, kinesthetic, interpersonal and intrapersonal intelligences [11]. As Lockhart points out,

> The only people who understand what is going on are the ones most often blamed and least often heard: the students. They say, "math class is stupid and boring," and they are right.

One of the chief problems is that math teachers are given high level directives to transform mathematics education, but not a lot of specific tools that exemplify these new ways of thinking. Dunn and Dunn's recommendations to help accommodate students' learning strengths [9], for example, asks teachers to consider how learners are affected by (1) immediate environment (sound, light, temperature, and design); (2) own emotionality (motivation, persistence, responsibility, and need for structure or flexibility); (3) sociological needs (self, pair, peers, team, adult, or varied); and (4) physical needs (perceptual strengths, intake, time, and mobility). This is almost antithetical to the traditional mathematics curriculum we all experienced, which is based on behaviorist rote learning, where students are given problem sets, are proscribed solution techniques, and are expected to repeat practice problems until mastery is achieved.

The consequences of this impoverishment in our educational curriculum are dire. The phenomena of math anxiety [1] and math avoidance [26] has been passed down through generations, and there is a cultural assumption that there are "math people" and "non-math people, "as if there could be some sort of genetic or anatomic disposition that makes one person more capable of understanding and executing mathematics than another.

Constructionism & Mathematics

Constructionist Seymour Papert was intimately concerned with trying to reform the problems of mathematics education [18]. Most math curricula are repurposed from traditional exercises to pursue correct answers to planned problems using narrow methods consistent with only a logical-mathematical intelligence. In the scheme of rigorous and overpopulated classrooms, educators have bandwidth for little else. This perpetuates a "relative poverty of culture in providing the materials to make the concept simple or concrete" [19]. Even more importantly, it prohibits any student investigation into exploring math data in ways that would be more relevant to personal understanding.

Papert's LOGO was originally targeted at the subject of math. Its creators argued that programming facilitated the acquisition of rigorous thinking and expression, gave students specific insights into how numbers, variable and functions work, motivated heuristics for problem solving, and helped students to analyze larger problems into smaller ones, among other things [10]. While many of the Digital Manipulatives [21] inspired by the LOGO/Lego research have extended LOGO's reach in teaching important and complex concepts such as emergent behaviors, dynamic systems, and programming paradigms, few of these projects in making programmable bricks, blocks, balls, beads and badges have addressed LOGO's original target subjects of geometry and algebra.

Art & Dance in Mathematics

Our work was inspired by Schaffer and Stern's *Math Dance*, a workbook with activities for interpreting beginning geometry concepts through performing choreography with gesture and movement and exploring the architecture of bodies in space (see Figure 2, next page) [23]. Similarly, instructors like Alexis Dekel of Pittsfield High School in Pittsfield, Massachusetts collaborated with a dance instructor and calculus students to create a Math Dance Team that concentrated on improvisation to create group choreography of advanced math concepts. Instead of a literal translation of a figure or equation, Pittsfield's Math Dance Team [7] embraces personal interpretations of how a math concept might look or feel in the body and communicates that understanding through movement that informs the choreography of the group; a unique interpretation of how embodied cognition can form mathematical understanding [14]. Our design sketches extend the movement lessons of Schaffer, Stern and Dekel by explicitly incorporating real-time visual feedback about position and movement through simultaneous camera capture and screen display. This allows the embodied activity to be studied further for its underlying math relationships while promoting students to be original and artistic to create visual designs. Interactive video installations like Ed Tannenbaum's *Recollections* series [25] allow participants to "paint" the screen with stacked layers of silhouettes made with body movement. Visualization tools such as the *Synchronous Objects* project from William Forsythe and Ohio State University's Advanced Computing Center for the Arts and Design [17], turned complicated dances into data in order to create virtual objects. Video filters and data visualization reveal relationships between dancers, objects, and the patterns they sculpt in space. Development of visualization and modeling tools that convert complicated gesture and movement into data are available for study [12, 17, 26]. The objects created from these systems, both virtual and fabricated, exploit the creative tendencies of bodies in motion while revealing some obstacles of transitioning between 3D and 2D space.

Methodology

This work employs design experimentation as a strategy for formative research. Our focus at this stage is to broaden the design space of possible mathematics curricula through speculative designs; formal evaluations of specific techniques will be conducted at a later point once the most promising designs have been identified and integrated into a more structured and controlled curriculum.

Our goal was to encourage mathematical exploration and discussion through rapid deployment of "interaction sketches" [4]. Each design is only a starting point with which to engage students and instructors, allowing them to construct their own words and actions in context of a creative activity. These activities uncover unique observations and insights within the group, and create connections about what changes and reactions each strategy might bring to future math curriculum. From the design evaluation standpoint, we studied the deployments through direct observation, video documentation and surveys when possible, and review of the creations/compositions students made using our interaction sketches.

Technology

Our technology choices were largely driven by a desire to maximize the future deployability and access of our designed tools. All of the experiments described in this paper were created and tested on a laptop computer using the Processing programming environment [20]. We employed libraries such as JMyron (an open-source computer-vision library which enables Processing towards motion detection, color tracking, and blob recognition) and TUIO (an application protocol interface to enable the recognition and tracking of fiducial markers) [3]. Occasionally, the use of separate USB web cameras and bluetooth keyboards were used to enable greater flexibility in spatial configuration of the designed activities. We also experimented with using a grid background in several of the experiments. All of these materials and tools are commercially available and inexpensive or free.

While we experimented in the lab with using wall projection and multi-camera set-ups, all of our sketch deployments used the laptop/web camera setup to maximize flexibility and minimize set-up time. These constraints also worked to maximize the future deployability of the designs we developed, as instructor preparation time is as limited as access to computers and peripheral technologies in many educational environments.

Participants

We solicited local mathematics teachers to volunteer one class period to try out each of our experiments. In total, we worked with one grade six class, one grade seven class, five high-school geometry or geometry/algebra classes, and one group of 12 teachers as part of a workshop at the San Francisco Exploratorium Learning Museum. In an effort to try out these sketches in many environments during a short deployment phase, we only made one visit per classroom. Hence, there was little opportunity for close collaboration or follow-up exercises with teachers.

Topics of Instruction

Our tool was targeted at the topics of geometry and algebra, specifically transformations of figures which are addressed at numerous points in elementary and high school

Figure 2. Adapting Schaeffer and Stern's *Math Dance* exercises to improvise choreography for geometric symmetries. The combination of dancers, camera, space, and virtual screen geometry keys opportunities for visualizing embodied experiences.

education, but handled as a specific focus in grades 8-10 in the California Public School System [2, 5, 24].

To develop understanding of transformations, the concepts the students and instructors are working on are: figure, preimage, image, variables, formulas, vectors, functions, equations, symmetry, isometries, area, vertices, planes, angles, rotation, reflection, translation, dilation, tessellation, approximation, models, similarity, congruence, patterns, sequences, series, and rules.

In the course of learning these concepts, students are expected to develop skills to do the following: design and conduct experiments to verify or disprove predictions; understand and make inferences based on the analysis of experimental results; recognize, create, extend, and generalize patterns, and understand, develop; and express rules describing patterns; use reading, listening, and observation skills to access and extract mathematical information from multiple, self-selected sources such as pictures, diagrams, physical models, oral narratives, and symbolic representations.

DESIGN EXPERIMENTS

The following are descriptions of the different interaction design sketches we made to test with students:

Patterned Poses

In Patterned Poses (Figure 3), captured frames from Math Angels (see this page, right column) were used as the root node to construct abstract patterns that enriched the visual exploration with elements like scale, color, and angles of rotation. This extended the discussion of transformations by starting with translations and reflective symmetry and visualizing effects from dilation and rotation. The Math Angels "shape" was turned into a silhouette to be repeated in different ways. The use of the body as a root form helped to personalize and make relevant the otherwise abstract forms; students were able to readily identify the original form and even whose body was imaged. One student survey response said, "Today was a '4' because we have found a new way of using geometry and art combined."

These example compositions are made using Illustrator scripts. A new digital prototype would use image processing software to create and isolate the silhouette. To create the desired aesthetic pattern, students could tune variables like scale, color, and angles of rotation, etc. via a screen interface. Most importantly, however, students can tune the overall shapes in their wallpaper visual by re-calibrating their body and repeating the capture process. The visual output simultaneously serves as a visual reference point to explain the finished transformations and an artistic exhibition of math relationships.

Math Angels

Inspired by the visual tools like Glowdoodle [22] and the construction of snow angels, Math Angels is an investigative tool using pixel recognition code to create multiple renderings of kinesthetic activities on a light-colored background surface. Students see a superimposition of the current feed of a web camera and previously-captured images; they can add the current frame to the stored stack by hitting a key on a USB keyboard (see Figure 1). By setting the alpha value of all the light colored pixels in the frame to zero, we create compositions that merge successive moments in time onto one screen image.

Math curriculum references transformations as an event that happens to a point, line, or shape on a plane, changing position, size, or orientation. By using the body as the "object" to be transformed, students have the opportunity to use layered screen captures to investigate what a transformation looks like during and after they try it kinesthetically. Two different versions placed the web camera pointed at a wall or directly overhead. This design factor encouraged students to attempt transformations by jumping in mid-air or by using more of their body without being limited by gravity. We constructed a grid of black lines on white vinyl to be in camera view. This dressing helped draw the best possible contrast for the alpha-value of pixels. It also helped students position themselves in space and served as a valuable visual marker for a math activity. During different tests we changed the orientation of the laptop screen to accommodate the performer or the audience.

Students were engaged to create examples of symmetry by themselves or by collaborating with others. The audience

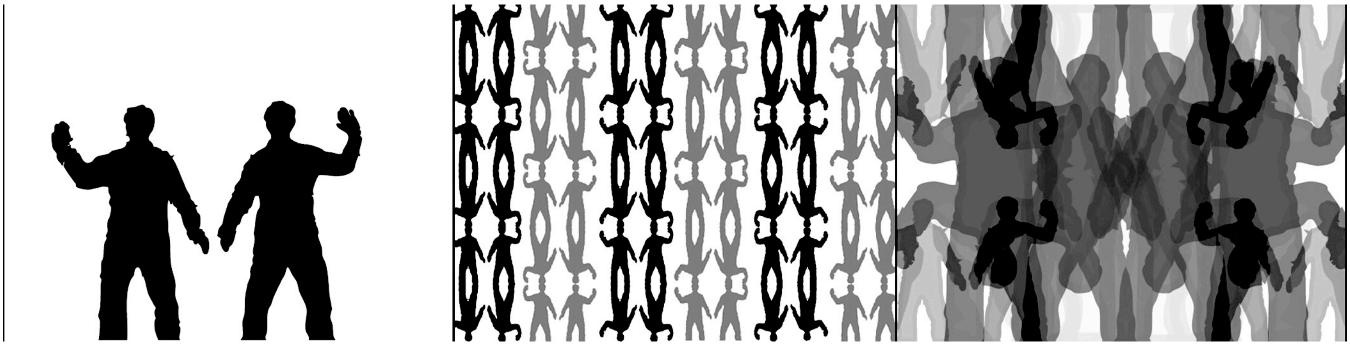

Figure 3. Using the silhouette images captured from video, Pattern Poses provides an artifact of embodied symmetry that becomes personal and artistic through tuning variables (color, scale, angles, tessellation patterns) in order to extend the dialogue about transformations further.

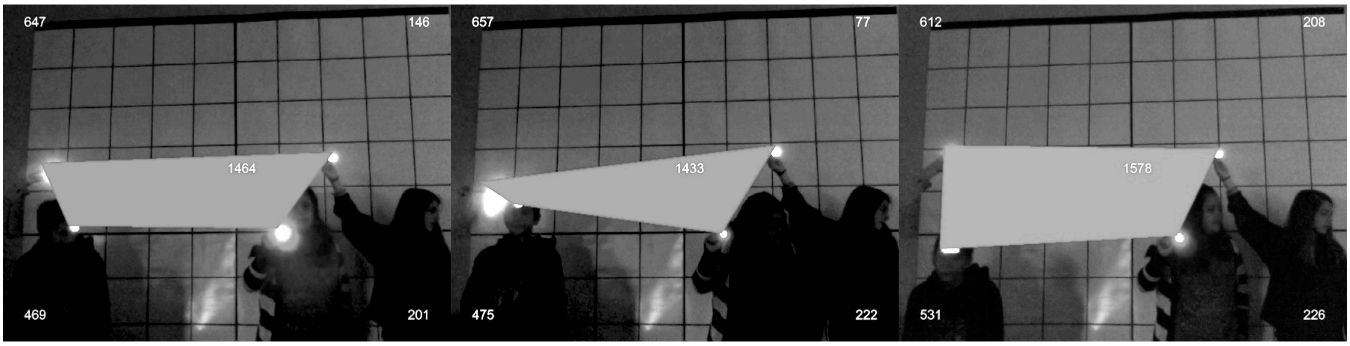

Figure 4. A group of sixth grade students create "Blob Shapes" on screen, using, flashlights and "blob detection" software to construct and manipulate the shapes, exploring congruency and symmetry using screen feedback and social interaction.

was also involved by directing the performers or suggesting new types of transforming movements. Teachers mostly observed students, managed audience members, and sometimes participated in creating different symmetries with students. Often times students completed a transformation like a translation or reflection and then added new layers of poses that created visual story elements instead of a specific math topic. 15 year old Gonzalo told us, "I never thought about geometry like this way."

Blob Shapes

Using Processing code and the JMyron external library, we track points of light and use them as vertices to construct simple polygons (triangles and quadrilaterals). This sketch was called Blob Shapes (Figure 4). Students position lights (we used handheld flashlights, headlamps and LED pucks) to different points on their body, or distribute lights amongst different players. Each light acts as vertex of the shape; when the player moves through the camera's field of vision virtual lines are drawn between the vertices to create the polygon. The shape of the polygon (its dimensions, perimeter, area) change along with the movement of the student's body (and the movement of the blobs). Students use the shape on the screen to inform their movements in order to achieve a desired transformation.

We used the grid background to better enable students to position their lights symmetrically. The screen display included numerical data about the length of each side of the polygon and the overall perimeter. The physical grid helped students make sense of the numbers on the virtual display. Additionally, the fill color (red) of the polygon on the screen increased opacity with an increase in the overall size of the polygon. This feedback was meant to help students maintain congruent shapes but also to observe how their body adjusted in response to screen output and vice-versa. Overall, this sketch was helpful in theorizing how body parts (especially the head, joints, and extremities) can be translated into vertices of polygons to reveal the geometry of the body. The animation of the shapes on screen also did much to help visualize the transitions that shapes make between transformations, similar to geometry software applications like Key Curriculum's Geometer's Sketchpad [12] but quite different from static renderings in math curriculum texts.

Video Kaleidoscope

Using a web camera and some customization of basic Processing code, we created different models of repeating video displays called Video Kaleidoscopes (Figure 5, next page). Some math textbooks use kaleidoscopic imagery and tessellation as visual examples[2, 24]. However, very few of the exercises lead students to create these patterns to form understanding of different symmetries. This sketch provided a way for teachers and students to discuss what they see on the page by participating directly in creating visual patterns that adhere to the same math rules.

Using a laptop with built-in camera, students take turns moving to affect the screen display. In several deployments we were able to project the screen display on an opposite wall for the entire class to view. Activating a button on a Bluetooth keypad saved the screen display as a static frame to be viewed later for reference. The video engaged participants to constantly try new orientations and combinations of position, depth, and movement. Students quickly brought in physical materials and hand-drawn designs from previous activities to augment the composition, creating new visual effects with shadows, reflection, and texture. While students did not immediately pick up on the mathematical aspects of this exercise, instructors were able to reference the Video Kaleidoscopes to discuss symmetry in further classroom discussion, and they could cite shared experiences, saying things like "remember when you walked from right to left and the pattern expanded outward." We received feedback that the expressive and artistic nature of this sketch helped different students engage: "we didn't write all the time and we had fun by learning as well."

REFLECTION

While we learned many things through our successive deployments, we wish to highlight the following lessons:

Introducing New Tools

While we wanted to motivate self-directed and open-ended student inquiry, we found that students would not engage with our tools without well-structured introductions.

Figure 5. Video Kaleidoscopes display updating tessellation (left), rotations (center) and reflections (right) that motivate movement in front of the camera; curiosity about how repeating images interact evolves into a structure for embodied geometry challenges.

Although each student session was limited to less than an hour, we found it worthwhile to explain and demonstrate how the tools worked, how they could control them, and what the results would be. For future development, these demonstrations should be performed by teachers themselves. We designed one written activity that mimicked the visual and content styles of a classroom lesson plan. This handout helped teachers make connections back to their curriculum while giving students an optional nudge but not constraining their creativity.

Early on we had questions whether we were working towards particular curriculum design or tools for specific learning concepts. In our workshop with math teachers at the Exploratorium, we got feedback that the latter was the best way to support and encourage the former; it was hard for teachers to generate these types of tools on their own even if they liked the ideas they embodied. Having the tools readily available (for free, they emphasized) would make it easier for them to rethink their curriculum to accommodate more visual and kinesthetic learning. We agree that this framework respects various teaching styles where technology tools are flexible enough to accommodate classroom diversity.

Social Engagement

While our one-laptop-per-classroom setup was driven largely by economic considerations and the ability to setup/ takedown quickly, we found that students sharing the capture tools motivated social engagement in many surprising and wonderful ways.

First, the sketches were treated as a performance activity. Just as video played on a screen or a guest speaker creating an experiment at the front of class demands attention, the sketches performed by one or a handful of students reinforced an actor/audience dynamic in the classroom. So, even with an expected amount of class commotion, the sketches kept the overall attention level high throughout the test visits. Some eager teachers who tried the sketches were indeed modeling creative behavior in a math activity for their students to follow.

Secondly, in spite of clear roles between performer and audience, students in the audience interacted with the sketch by offering advice or new ideas to the performer, supplying positive feedback to success as well as articulating critical responses to the quality of a performer's transformation. In this way the entire class behaved collaboratively, brainstorming new ways to use the tools. Figure 6 (next page) is just one example how students used the Math Angels program to build moral narratives (such as the gunfight with innocent bystanders on top of the framework of creating reflective symmetry), invent special effects (such as capturing successive frames mid-jump to create an illusion of floating while showing off rotational symmetry around a cardboard tube "axis") and experiment with layering and depth (by standing closer to the camera to create a "friendship collage" while exhibiting a dilation transformation).

Third, the verbal and kinesthetic communication about where to stand, how to pose, how to hold things, and what was being controlled extended the vocabulary used to describe the mathematical relationships in the sketches. For instance, references to "raise your arm a little more" to adjust vertices, "move back one step" explains action on the x-axis, and "face front…now turn around" portrays a specific rotation on the body's axis. Students used expressions like: "Make a class star!," "I can hang it from my head and make a pentagon," and "You can try to put someone inside of a shape."

Abstraction

Our experiments with abstracting the screen images (for example, in Patterned Poses) was an attempt to make the tools more artistic while simultaneously engaging with math. We could challenge students to think about shapes in a less structured way by abstracting where the initial forms came from through silhouetting.

Although our experiments are far from conclusive with regard to abstraction, we underestimated how much the students *enjoyed* being able to see their friends faces and forms on the images we captured. They were constantly looking for recognizable aspects and were very concerned about who the images "belonged to." At the same time, it's important to note that such a literal capture of body likeness was distressing for some students who were engaged as

Figure 6. Students using "Math Angels" sketches to construct narratives with reflections(left), generate special effects with rotations and translations (center), and to explore identity and social engagement using dilation (right).

audience but were adamant about not performing. It was an important reminder of the influence of camera/screen technology and the psychology of self image in young adults.

In our session with math teachers at the Exploratorium we found that the teachers preferred when the sketches included numbers on the interface; it made them feel more confident the activities were appropriate for a math class. Their suggestions were often tied to specific lessons or topics that they covered, such as grid relationships and graphing functions. We were heartened to find that many of the sketches could be adapted to a wide variety of lesson plans. But we also wonder whether the specific adaptations should be made by instructor rather than be built into the tools themselves.

Narrative

We were pleasantly surprised by the way that the performative aspects of our tool led directly to storytelling, an uncommon outcome in mathematics curriculum. The examples in Figure 6 all had strong story-telling elements that also revealed surprising moral components. In one Math Angels composition, students processed the concept of reflective symmetry in their performance through both their characters' *personality* as well as *body position*. Compositions made with Video Kaleidoscope became a lot more interesting to students when one of them posed a challenge–for example, to try to create a triangle using the six-sided reflection tool. By combining the faculties of the body with the creative mind to investigate math, students have more leverage to use intuition and self-expression to create personal agency and compelling stories as references for understanding concepts.

FUTURE WORK

This series of interaction design sketches illustrates and tests our basic ideas about combining kinesthetic, visual, social, expressive and mathematic learning. Moving forward we would like to develop these designs into a more systematic integration with curriculum. It is necessary to explore more intuitive screen and wireless controller interface as well as possibilities for projecting digital displays onto physical spaces (wall, floors) to facilitate embodied thinking into kinesthetic action more immediately. Teachers at the Exploratorium effectively used masking techniques with white paper to affect alpha values on Math Angels screens, a type of "visual hack" that has huge potential for manipulating patterns.

Additionally, it would be beneficial to build in screen prompts to encourage participation through basic "training" exercises. In addition, its necessary to hold sustained, formal evaluations with specific math classes over the course of a school year to understand the longer-term effects of these designs and better adapt these tools so they can be readily taken up by mathematics instructors teaching in varied contexts. In this way, it's essential to formulate a model of assessment that supports math comprehension but does not discount or undervalue the creative aspects of these interactions.

CONCLUSION

In Math Propulsion, we present a different vision of how computation can be used to engage math learners with spatial relationships of geometry. Instead of using the computer's language to interact with numbers, students translate body movement into the flexible virtual worlds of math data through a performance with interactive elements in the classroom environment. In this sense, students develop math knowledge in real life as well as on screen. This more personal exploration of math and movement becomes a learning model for students in other aspects of their world.

We think that our tool embodies valuable techniques that researchers recommend to combat the perceived difficulty and irrelevance of math. We hope that widespread dissemination of tools such as ours might inspire math instructors to think about how they, too, could engage different learning styles and intelligences, and motivate students to be creative, take risks, and engage in social discussion around mathematical concepts.

One of our key motivations was to use kinesthetic action as a way to make the abstract concrete through the embodied mind. But our experiments showed us something more. In the end, the most effective simulations will not just supply a more concrete way of exploring abstract math. The interactions should be expressive, beautiful, and elicit a "it was fun" reaction, which we heard often in classrooms. In this sense, this inquiry will contribute to the growing definition of successful interaction design, where designs encourage self-expression and creativity, not just utility and efficiency. If a combination of curriculum and technology reveal new avenues for kinesthetic learning as effective problem-solving strategies, creativity and intuition will gain currency in the world of education and industry.

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